Non-linear Distortion Can Eliminate Harmonics

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September 2013

For a sinusoidal input signal (which contains only the fundamental frequency) it is well known that passing the signal through a non-linear system will always add harmonic frequency components into the output signal.

It is interesting to find that for a harmonically-rich input signal it is possible for a non-linear system to remove harmonics, giving a pure sinusoid at the output!

This can be demonstrated by a simple example (illustrated in figure 1):

The input signal is the (harmonically-rich) triangular wave, defined as:

$$V_{in} = \omega t \quad \{\text{for } -\pi/2 < \omega t \le \pi/2\}$$
$$= \pi - \omega t \quad \{\text{for } \pi/2 < \omega t \le 3\pi/2\}$$
etc.

where ω is the angular frequency in radians
$\omega = 2\pi f$ (where f is the frequency in hertz)

The non-linear system, shown in figure 2, has the following input-to-output relationship:

$$V_{out} = -1 \quad \{\text{ for } V_{in} \le -\pi/2 \}$$
$$= \sin (V_{in}) \quad \{\text{ for } -\pi/2 < V_{in} < \pi/2\}$$
$$= 1 \quad \{\text{ for } V_{in} \ge \pi/2 \}$$

Applying the non-linearity to the input triangular wave the output becomes a pure sinusoid:

$$V_{out} = \sin (\omega t)$$

This is very easy to see in the first quadrant ($0 \le \omega t \le \pi/2$) and the result for the remaining time periods ($\omega t < 0$ and $\omega t > \pi/2$) can also be calculated, but is readily apparent by considering symmetry.

Limitations and Implications:

The above example is a special case in which the input triangular wave has a peak-to-peak amplitude that exactly matches the non-linearity. However, the non-linear input-to-output relationship of an over-driven amplifier can be similar to the function defined in the example. It is reminiscent of an over-driven centre-biased pentode preamp stage. On the other hand, a guitar signal is no closer to a triangular wave than it is to a sine wave. If nothing else, this example shows that in the right circumstances a distorted output can be a 'smoother' sound than the original input signal.

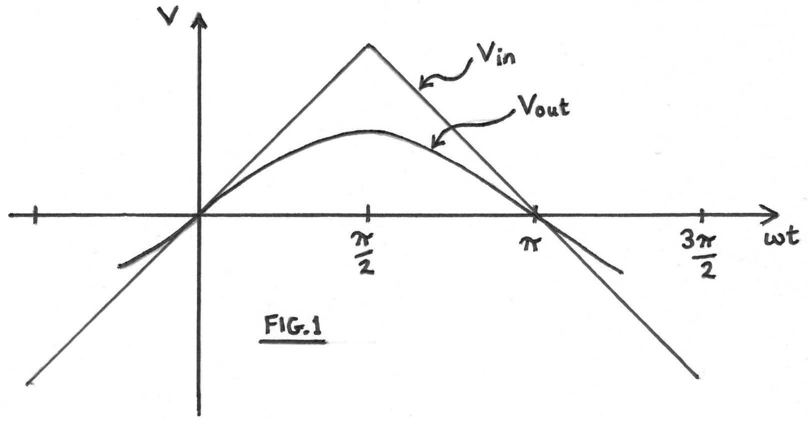

FIG. 1

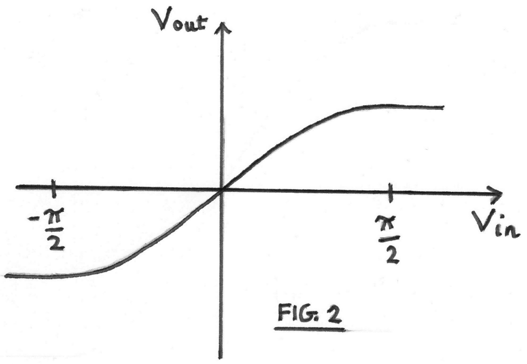

FIG. 2